# SECTION 13 - METHOD OF DETERMINING PERCENTAGE WITHIN LIMITS (FAA 110) 

## 13-1 GENERAL

This section describes the Method of Determining the Percentage Within Limits (PWL) used to calculate adjusted pay factors for pavement. PWL determination shall be made in accordance with the FAA Standard Specification 110 as follows:

## SECTION 110 - METHOD OF ESTIMATING PERCENTAGE OF MATERIAL WITHIN SPECIFICATION LIMITS (PWL)

110-1.1 GENERAL. When the specifications provide for acceptance of material based on the method of estimating percentage of material within specification limits (PWL), the PWL will be determined in accordance with this section. All test results for a lot will be analyzed statistically to determine the total estimated percent of the lot that is within specification limits. The PWL is computed using the sample average $(X)$ and sample standard deviation $\left(S_{n}\right)$ of the specified number ( $n$ ) of sublots for the lot and the specification tolerance limits, $L$ for lower and $U$ for upper, for the particular acceptance parameter. From these values, the respective Quality index(s), $Q_{L}$ for Lower Quality Index and/or $Q_{U}$ for Upper Quality Index, is computed and the PWL for the lot for the specified $n$ is determined from Table 1.

There is some degree of uncertainty (risk) in the measurement for acceptance because only a small fraction of production material (the population) is sampled and tested. This uncertainty exists because all portions of the production material have the same probability to be randomly sampled. The Contractor's risk is the probability that material produced at the acceptable quality level is rejected or subjected to a pay adjustment. The Owner's risk is the probability that material produced at the rejectable quality level is accepted.

It is the intent of this section to inform the Contractor that, in order to consistently offset the Contractor's risk for material evaluated, production quality (using population average and population standard deviation) must be maintained at the acceptable quality specified or higher. In all cases, it is the responsibility of the Contractor to produce at quality levels that will meet the specified acceptance criteria when sampled and tested at the frequencies specified.

110-1.2 METHOD FOR COMPUTING PWL. The computational sequence for computing PWL is as follows:
a. Divide the lot into $n$ sublots in accordance with the acceptance requirements of the specification.
b. Locate the random sampling position within the sublot in accordance with the requirements of the specification.
c. Make a measurement at each location, or take a test portion and make the measurement on the test portion in accordance with the testing requirements of the specification.
d. Find the sample average ( X ) for all sublot values within the lot by using the following formula:
$X=\left(x_{1}+x_{2}+x_{3}+\ldots x_{n}\right) / n$
Where: $\quad X \quad=$ Sample average of all sublot values within a lot
$x_{1}, x_{2}=$ Individual sublot values
$n \quad=$ Number of sublots
e. Find the sample standard deviation (Sn) by use of the following formula:
$S_{n}=\left[\left(d_{1}^{2}+d_{2}^{2}+d_{3}^{2}+\ldots d_{n}^{2}\right) /(n-1)\right]^{1 / 2}$
Where:

| $S_{n}$ | $=$ |
| :--- | :--- |
| $d_{1}, d_{2}, \ldots$ | $=$Sample standard deviation of the number of sublot <br> values in the set |
| $n$ | $=$Deviations of the individual sublot values $x_{1}, x_{2}, \ldots$ <br> from the average value $X$ |
| that is: $d_{1}=\left(x_{1}-X\right), d_{2}=\left(x_{2}-X\right) \ldots d_{n}=\left(x_{n}-X\right)$ |  |
| Number of sublots |  |

f. For single sided specification limits (i.e., L only), compute the Lower Quality Index $Q_{L}$ by use of the following formula:
$Q_{L}=(X-L) / S_{n}$
Where: $L=$ specification lower tolerance limit
Estimate the percentage of material within limits (PWL) by entering Table 1 with $Q_{L}$, using the column appropriate to the total number ( $n$ ) of measurements. If the value of $Q_{L}$ falls between values shown on the table, use the next higher value of PWL.
g. For double sided specification limits (i.e., $L$ and $U$ ), compute the Quality Indexes $Q_{L}$ and $Q_{U}$ by use of the following formulas:
$Q_{L}=(X-L) / S n \quad$ and $\quad Q_{U}=(U-X) / S n$
Where: $\quad L$ and $U \quad=$ specification lower and upper tolerance limits

Estimate the percentage of material between the lower ( $L$ ) and upper ( $U$ ) tolerance limits (PWL) by entering Table 1 separately with $Q_{L}$ and $Q_{U}$, using the column appropriate to the total number (n) of measurements, and determining the percent of material above $P_{L}$ and percent of material below $P_{U}$ for each tolerance limit. If the values of $Q_{L}$ fall between values shown on the table, use the next higher value of $P_{L}$ or $P_{U}$. Determine the PWL by use of the following formula:
$P W L=\left(P_{U}+P_{L}\right)-100$
Where: $\quad P_{L}=$ percent within lower specification limit
$P_{U}=$ percent within upper specification limit

## EXAMPLE OF PWL CALCULATION

Project: Example Project
Test Item: Item P-401, Lot A.
a. Determination for Mat Density
(1) Density of four random cores taken from Lot A.

A-1 96.60
A-2 97.55
A-3 99.30
A-4 98.35
$n=4$
(2) Calculate average density for the lot.
$X=(x 1+x 2+x 3+\ldots x n) / n$
$X=(96.60+97.55+99.30+98.35) / 4$
$X=97.95$ percent density
(3) Calculate the standard deviation for the lot.

$$
\begin{aligned}
S n= & {\left[\left((96.60-97.95)^{2}+(97.55-97.95)^{2}+(99.30-97.95)^{2}\right.\right.} \\
& \left.\left.\left.+(98.35-97.95)^{2}\right)\right) /(4-1)\right]^{1 / 2} \\
S n= & {[(1.82+0.16+1.82+0.16) / 3]^{1 / 2} } \\
S n= & 1.15
\end{aligned}
$$

(4) $\quad$ Calculate the Lower Quality Index $Q_{L}$ for the lot $(L=96.3)$.

$$
\begin{aligned}
Q_{L} & =(X-L) / S n \\
Q_{L} & =(97.95-96.30) / 1.15 \\
Q_{L} & =1.4348
\end{aligned}
$$

(5) Determine PWL by entering Table 1 with $Q_{L}=1.44$ and $n=4$.
$P W L=98$

## b. PWL Determination for Air Voids

(1) Air Voids of four random samples taken from Lot A.
A-1 5.00

A-2 3.74
A-3 2.30
A-4 3.25
(2) Calculate the average air voids for the lot.

$$
\begin{aligned}
& X=\left(x_{1}+x_{2}+x_{3}+. . n\right) / n \\
& X=(5.00+3.74+2.30+3.25) / 4 \\
& X=3.57 \text { percent }
\end{aligned}
$$

(3) Calculate the standard deviation $S_{n}$ for the lot.

$$
\begin{aligned}
S_{n}= & {\left[\left((3.57-5.00)^{2}+(3.57-3.74)^{2}+(3.57-2.30)^{2}\right.\right.} \\
& \left.+(3.57-3.25))^{2} /(4-1)\right]^{1 / 2} \\
S_{n}= & {[(2.04+0.03+1.62+0.10) / 3]^{1 / 2} } \\
S_{n}= & 1.12
\end{aligned}
$$

(4) Calculate the Lower Quality Index $Q_{L}$ for the lot $(L=2.0)$.

$$
\begin{aligned}
& Q_{L}=(X-L) / S_{n} \\
& Q_{L}=(3.57-2.00) / 1.12 \\
& Q_{L}=1.3992
\end{aligned}
$$

(5) Determine $P_{L}$ by entering Table 1 with $Q_{L}=1.40$ and $n=4$.

$$
P_{L}=97
$$

(6) $\quad$ Calculate the Upper Quality Index $Q_{U}$ for the lot $(U=5.0)$.

$$
\begin{aligned}
& Q_{U}=(U-X) / S_{n} \\
& Q_{U}=(5.00-3.57) / 1.12 \\
& Q_{U}=1.2702
\end{aligned}
$$

(7) Determine $P_{U}$ by entering Table 1 with $Q_{U}=1.27$ and $n=4$.

$$
P_{U}=93
$$

(8) Calculate Air Voids PWL.
$P W L=\left(P_{L}+P_{U}\right)-100$
$P W L=(97+93)-100=90$

TABLE 1. TABLE FOR ESTIMATING PERCENT OF LOT WITHIN LIMITS (PWL)

|  | Positive Values of $Q$ |  |
| :--- | :--- | :---: |


| Percent Within Limits (PWL), <br> $P_{\mathrm{L}}$, and $P_{U}$ | $n=3$ | $n=4$ | $n=5$ | $n=6$ | $n=7$ | $n=8$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |


| 99 | 1.1541 | 1.4700 | 1.6714 | 1.8008 | 1.8888 | 1.9520 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 98 | 1.1524 | 1.4400 | 1.6016 | 1.6982 | 1.7612 | 1.8053 |
| 97 | 1.1496 | 1.4100 | 1.5427 | 1.6181 | 1.6661 | 1.6993 |
| 96 | 1.156 | 1.380 | 1.4897 | 1.5497 | 1.5871 | 1.6127 |


| 96 | 1.1456 | 1.3800 | 1.4897 | 1.5497 | 1.5871 | 1.6127 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 95 | 1.1405 | 1.3500 | 1.4407 | 1.4887 | 1.5181 | 1.5381 |
| 94 | 1.1342 | 1.3200 | 1.3946 | 1.4329 | 1.4561 | 1.4716 |
| 93 | 1.1269 | 1.2900 | 1.3508 | 1.381 | 1.3991 | 1.4112 |


| 93 | 1.1269 | 1.2900 | 1.3508 | 1.3810 | 1.3991 | 1.4112 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 92 | 1.1184 | 1.2600 | 1.3088 | 1.3323 | 1.3461 | 1.3554 |
| 91 | 1.1089 | 1.2300 | 1.2683 | 1.2860 | 1.2964 | 1.3032 |
| 90 | 1.0982 | 1.2000 | 1.2290 | 1.2419 | 1.2492 | 1.2541 |


| 90 |  |
| :---: | :---: |
| 89 |  |
| 88 |  |


| 87 |  |
| :--- | :--- |
| 86 |  |
| 85 |  |

TABLE 1. TABLE FOR ESTIMATING PERCENT OF LOT WITHIN LIMITS (PWL)

|  | Positive Values of $Q$ |  |  |
| :--- | :--- | :--- | :--- |


| Percent Within Limits (PWL), <br> $\boldsymbol{P}_{\mathbf{L}}$, and $\boldsymbol{P}_{\mathbf{U}}$ |  |
| :---: | :---: |
| 65 |  |


| TABLE 1. TABLE FOR ESTIMATING PERCENT OF LOT WITHIN LIMITS (PWL) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Positive Values of Q |  |  |  |  |  |
| Percent Within Limits (PWL), $P_{L}$, and $P_{U}$ | $n=3$ | $n=4$ | $n=5$ | $n=6$ | $n=7$ | $n=8$ |
| 28 | -0.7360 | -0.6600 | -0.6316 | -0.6176 | -0.6095 | $-0.6044$ |
| 27 | -0.7636 | -0.6900 | -0.6617 | -0.6477 | -0.6396 | -0.6344 |
| 26 | -0.7904 | -0.7200 | -0.6921 | -0.6781 | -0.6701 | -0.6649 |
| 25 | -0.8165 | -0.7500 | -0.7226 | -0.7089 | -0.7009 | -0.6958 |
| 24 | -0.8417 | -0.7800 | -0.7535 | -0.7401 | -0.7322 | -0.7271 |
| 23 | -0.8662 | -0.8100 | -0.7846 | -0.7716 | -0.7640 | -0.7590 |
| 22 | -0.8897 | -0.8400 | -0.8160 | -0.8036 | -0.7962 | -0.7915 |
| 21 | -0.9124 | -0.8700 | -0.8478 | -0.8360 | -0.8291 | -0.8245 |
| 20 | -0.9342 | -0.9000 | -0.8799 | -0.8690 | -0.8625 | -0.8583 |
| 19 | -0.9550 | -0.9300 | -0.9123 | -0.9025 | -0.8966 | -0.8928 |
| 18 | -0.9749 | -0.9600 | -0.9452 | -0.9367 | -0.9325 | -0.9281 |
| 17 | -0.9939 | -0.9900 | -0.9785 | -0.9715 | -0.9672 | -0.9643 |
| 16 | -1.0119 | -1.0200 | -1.0124 | -1.0071 | -1.0037 | -1.0015 |
| 15 | -1.0288 | -1.0500 | -1.0467 | -1.0435 | -1.0413 | -1.0399 |
| 14 | -1.0448 | -1.0800 | -1.0817 | -1.0808 | -1.0800 | -1.0794 |
| 13 | -1.0597 | -1.1100 | -1.1173 | -1.1191 | -1.1199 | -1.1204 |
| 12 | -1.0736 | -1.1400 | -1.1537 | -1.1587 | -1.1613 | -1.1630 |
| 11 | -1.0864 | -1.1700 | -1.1909 | -1.1995 | -1.2043 | -1.2075 |
| 10 | -1.0982 | -1.2000 | -1.2290 | -1.2419 | -1.2492 | -1.2541 |
| 9 | -1.1089 | -1.2300 | -1.2683 | -1.2860 | -1.2964 | -1.3032 |
| 8 | -1.1184 | -1.2600 | -1.3088 | -1.3323 | -1.3461 | -1.3554 |
| 7 | -1.1269 | -1.2900 | -1.3508 | -1.3810 | -1.3991 | -1.4112 |
| 6 | -1.1342 | -1.3200 | -1.3946 | -1.4329 | -1.4561 | -1.4716 |
| 5 | -1.1405 | -1.3500 | -1.4407 | -1.4887 | -1.5181 | -1.5381 |
| 4 | -1.1456 | -1.3800 | -1.4897 | -1.5497 | -1.5871 | -1.6127 |
| 3 | -1.1496 | -1.4100 | -1.5427 | -1.6181 | -1.6661 | -1.6993 |
| 2 | -1.1524 | -1.4400 | -1.6016 | -1.6982 | -1.7612 | -1.8053 |
| 1 | -1.1541 | -1.4700 | -1.6714 | -1.8008 | -1.8888 | -1.9520 |

## END OF SECTION 110

## 13-2 METHOD OF MEASUREMENT AND BASIS OF PAYMENT

Work described in this section shall be considered incidental to other pay items and there shall be no separate payment.

## END OF SECTION 13

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